King Fahd University of Petroleum & Minerals

College of Computer Science and Engineering

Information and Computer Science Department

Second Semester 172 (2017/2018)

ICS 202 – Data Structures
Major Exam 2
Time: 120 minutes

Name: _____

ID# | | | | | | | | | |

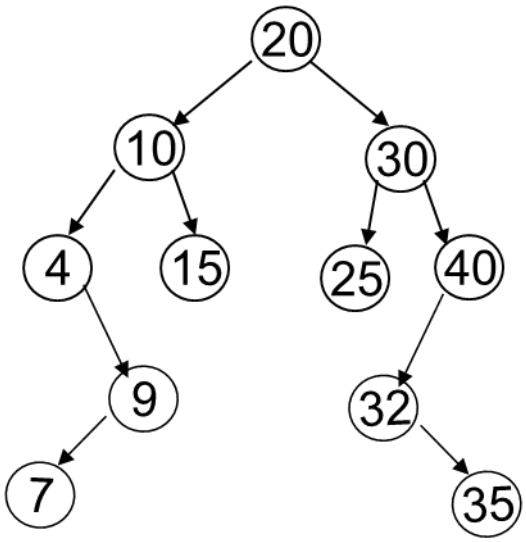
Section 01 Dr. Wasfi
Section 02 Dr. Emad

Question #	Max Marks	Marks Obtained
1	20	
2	10	
3	10	
4	10	
5	15	
6	15	
7	20	
Total	100	

Instructions

1. Write your name and ID in the respective boxes above and circle your section.
2. This exam consists of 7 pages, including this page, containing 7 questions.
3. You have to answer all 7 questions.
4. The exam is closed book and closed notes. No calculators or any helping aids are allowed.
5. Make sure you turn off your mobile phone and keep it in your pocket if you have one.
6. The questions are not equally weighed.
7. The maximum number of points for this exam is 100.
8. You have exactly 90 minutes to finish the exam.
9. Make sure your answers are readable.
10. If there is no space on the front of the page, feel free to use the back of the page. Make sure you indicate this in order not to miss grading it.

Q.1 (20 points: 2x10): Binary Trees Consider the following tree:



a) List the nodes of the left subtree of the root:
10, 4, 15, 9, 7

b) List the ancestors of node 15:
15, 10, 20

c) List the proper ancestors of node 15:
10, 20

d) List the proper descendants of node 30:
25, 40, 32, 35

e) What is the degrees of nodes 20, 15, and 32 respectively?
2, 0, 1

f) List all internal nodes of the above tree:
20, 10, 30, 4, 40, 9, 32

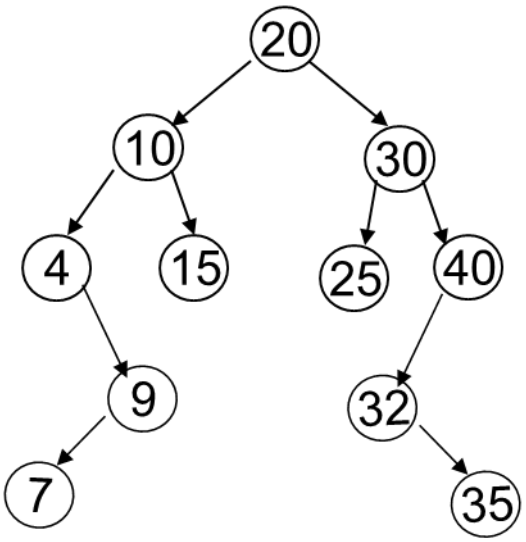
g) Are nodes 15 and 25 siblings?
No

h) What are the levels of nodes 7, 15, and 20 respectively?
5, 3, 1

i) What are the heights of nodes 30, 25, and 32 respectively?
4, 1, 2

j) Is the above tree full or complete?
No. No.

Q.2 (10 points): Binary Search Trees Consider the following tree:



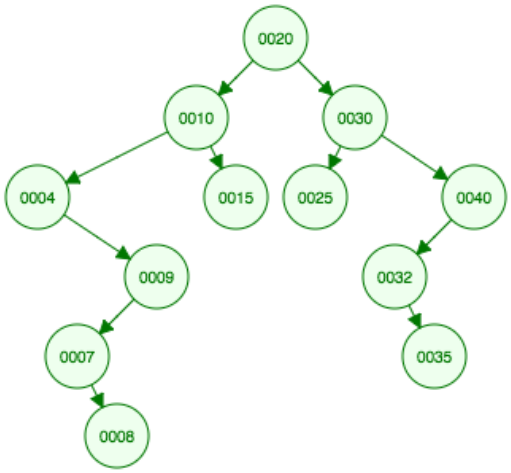
a. (4 points) What is the output of the pre-order traversal of the above tree starting from the root?

20, 10, 4, 9, 7, 15, 30, 25, 40, 32, 35

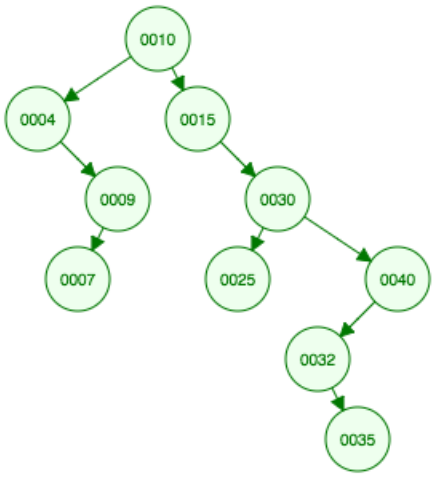
b. (3 points) Insert the node with key 8 into the above binary search tree.

c. (3 points) Delete the root node (20) by merging from the above binary search tree.

b



c



Q.3 (10 points): Consider the following recursive algorithm `weird`, where we are interested in finding its time complexity in terms of the number of element additions that are carried out in line numbers 6 and 11:

```
1. static int weird (int [] A, int n) {
2.   if (n <= 0)
3.      return 1;
4.   else {
5.      if (n is even) {
6.         return A[n] + weird(A, n/2);
7.      } else
8.        {
9.           int k=0;
10.          for i = 1 to n
11.             k = k + A[i];
12.          return k * weird(A, n-2);
13.       }
14.  }
```

a. (5 points) Derive the recurrence equation describing the **best case** time complexity of the algorithm.

$$T(n) = \begin{cases} 0 & n \leq 0 \\ T\left(\frac{n}{2}\right) + 1 & n > 0 \end{cases}$$

b. (5 points) Derive the recurrence equation describing the **worst case** time complexity of the algorithm.

$$T(n) = \begin{cases} 0 & n \leq 0 \\ T(n-2) + n & n > 0 \end{cases}$$

Q.4 (10 points): Solve the following recurrence relation function:

$$T(n) = \begin{cases} 8T\left(\frac{n}{2}\right) + n^2 & n > 1 \\ 1 & n = 1 \end{cases}$$

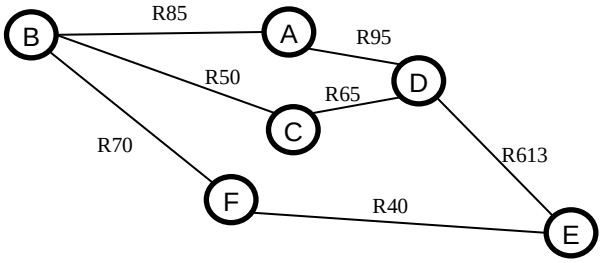
$$T(n) = 8T\left(\frac{n}{2}\right) + n^2$$
$$= 8\left[8T\left(\frac{n}{2^2}\right) + \left(\frac{n}{2}\right)^2\right] + n^2$$
$$= 8^2 T\left(\frac{n}{2^2}\right) + 2n^2 + n^2$$
$$= 8^2\left[8T\left(\frac{n}{2^3}\right) + \left(\frac{n}{2^2}\right)^2\right] + 2n^2 + n^2$$
$$= 8^3 T\left(\frac{n}{2^3}\right) + 8^2\left(\frac{n^2}{4^2}\right) + 2n^2 + n^2$$
$$= 8^3 T\left(\frac{n}{2^3}\right) + 4n^2 + 2n^2 + n^2$$
$$\vdots$$
$$= 8^k T\left(\frac{n}{2^k}\right) + 2^{k-1} n^2 + 2^{k-2} n^2 + \cdots + 2^0 n^2$$
$$= 8^k T\left(\frac{n}{2^k}\right) + n^2 \sum_{j=0}^{k-1} 2^j$$

Expansion will stop when $\frac{n}{2^k} = 1$

i.e. $n = 2^k \Leftrightarrow k = \lg n$

$$T(n) = 8^{\lg n} T(1) + n^2 \sum_{j=0}^{\lg n - 1} 2^j$$
$$= 2^{3\lg n} + n^2 \cdot \frac{2^{\lg n - 1 + 1} - 1}{2 - 1}$$
$$= 2^{\lg n^3} + n^2 (n - 1)$$
$$= n^3 + n^3 - n^2 = 2n^3 - n^2.$$

Q.5 (15 points): Consider the following undirected graph that represents cities with the names of some roads connecting between them:



A. (5 points) Write down the incidence matrix representation of the above graph.

	R40	R50	R65	R70	R85	R95	R613
A	0	0	0	0	1	1	0
B	0	1	0	1	1	0	0
C	0	1	1	0	0	0	0
D	0	0	1	0	0	1	1
E	1	0	0	0	0	0	1
F	1	0	0	1	0	0	0

B. (10 points) Using the incidence matrix graph representation, what is the time complexity of the following operations, in the worst case, when applied on a graph with n vertices and m edges:

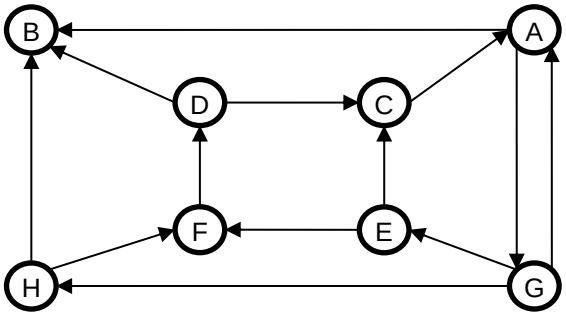
a. (5 points) Finding whether two cities v_1 and v_2 are connected by a direct road. Justify your answer.

O(m). We need to check the rows of v_1 & v_2 whether there is a column in which both are 1.

b. (5 points) Finding the degree of a city v (the number of different roads v is connected to). Justify your answer.

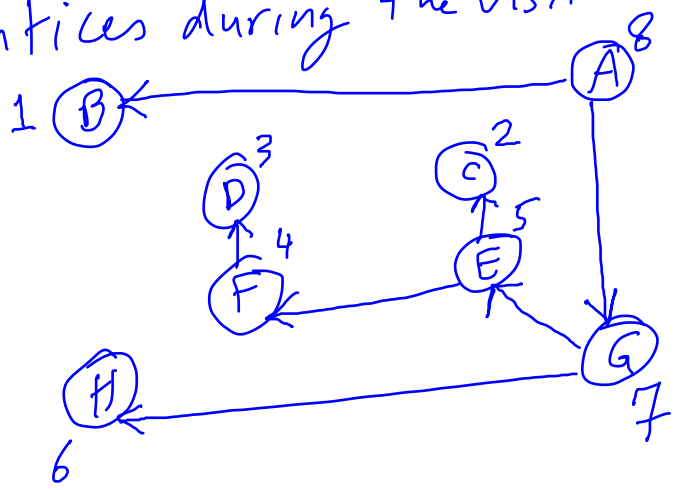
O(m). We need to add the values of the row of v to get the degree.

Q.6 (15 points): Consider the following directed graph:

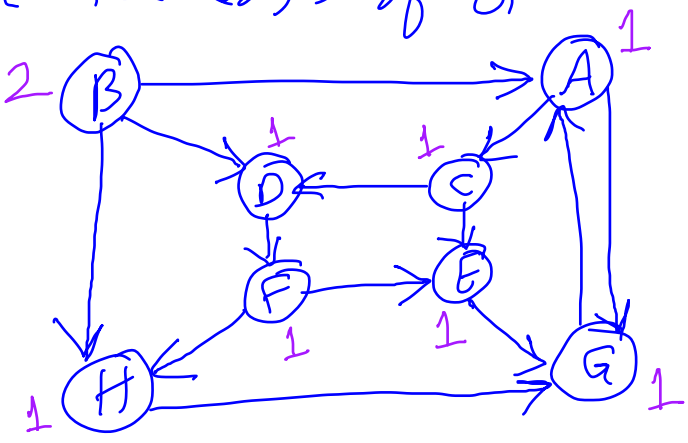


Run the SCC Algorithm to find the strongly connected components of the graph. Show clearly all the steps.

1. Run post-order depth first traversal, numbering the vertices during the visit.



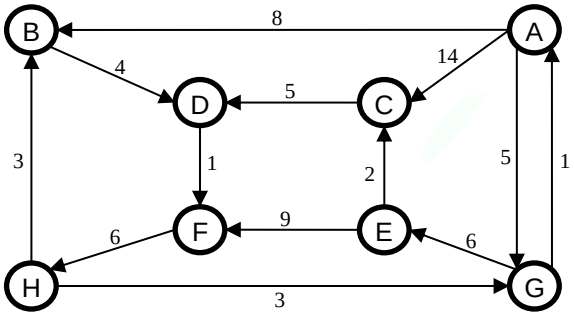
2. Reverse the edges of G.



3. Run depth-first traversal, numbering each component, starting from highest number to lowest

4. 2 SCC's: {B}, {A, C, D, E, F, G, H}

Q.7 (20 points): Consider the following directed weighted graph:



Run Dijkstra's algorithm to solve the single source shortest path problem by filling the table below starting from vertex A. Show the shortest path tree at the end.

Pass										Shortest Distance	Predecessor
Active Vertex	initially	A	G	B	E	D	C	F	H		
A	0									0	
B	∞	8	8							8	A
C	∞	14	14	14	13	13				13	E
D	∞	∞	∞	12	12					12	B
E	∞	∞	11	11						11	G
F	∞	∞	∞	∞	20	13	13			13	D
G	∞	5								5	A
H	∞	∞	∞	∞	∞	∞	∞	19		19	F